Lecture 3: Programming ZKPs

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Zero Knowledge Proofs

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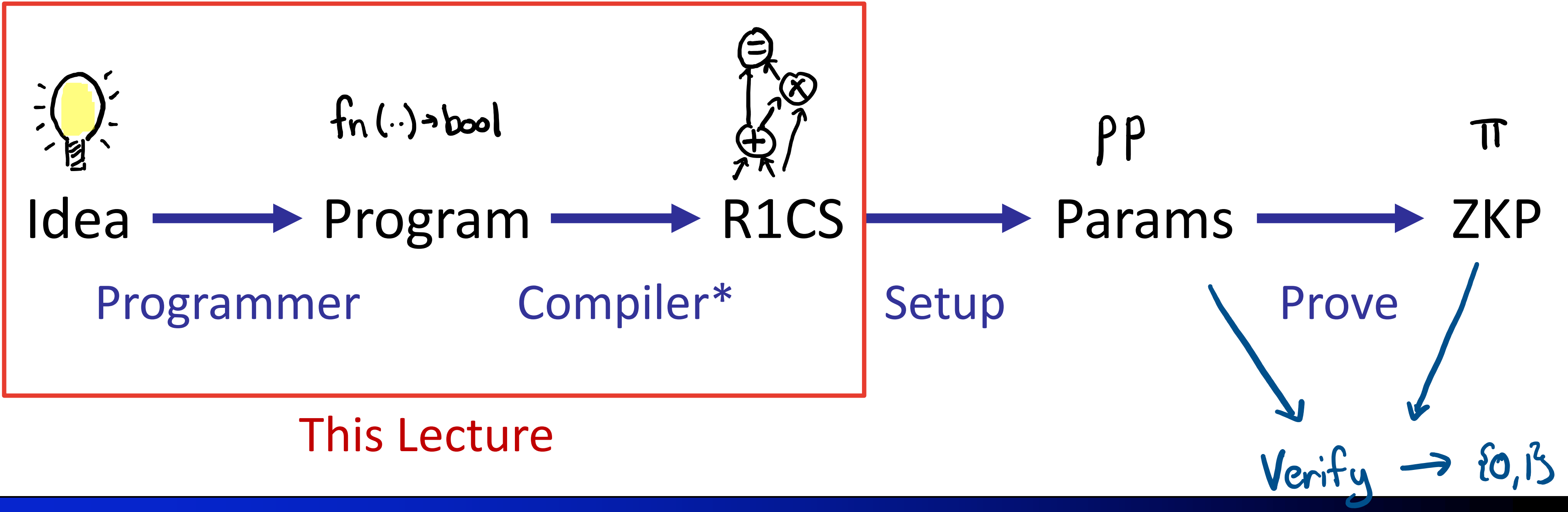
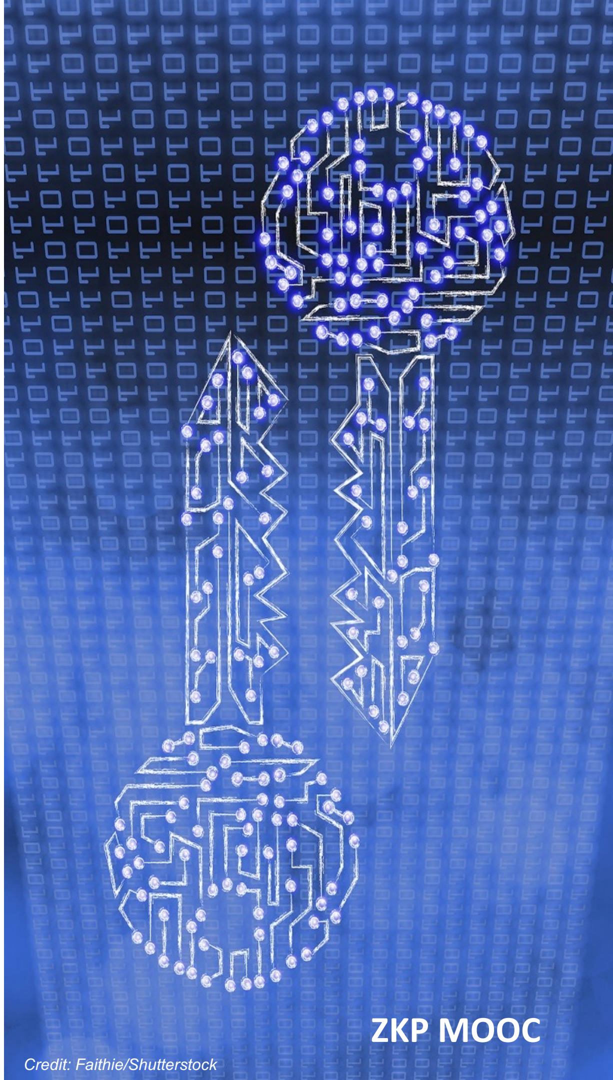
Using a ZKP


fn(..)→bool
pp
π
Idea
Program
R1CS
Params
ZKP
Programmer
Compiler*
Setup
Prove
This Lecture
Verify → {0,1}

This Lecture

1. Big Picture: ZKP programmability
2. Using an HDL (+ tutorial)
3. Using a library (+ tutorial)
4. Using a compiler (+ tutorial)
5. An overview of prominent ZKP toolchains

ZKP Programmability



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Recap: ZKPs for a predicate ϕ

- Prover knows ϕ, x, w
- Verifier knows ϕ, x
- Proof π shows that $\phi(x, w)$ holds
  - but does not reveal w

- Key Question: what can ϕ be?

What is ϕ?

ϕ in theory

- w is a factorization of integer x
- w is the secret key for public key x
- w is the credential for account x
- w is a valid transaction

ϕ in practice

- ϕ is an "arithmetic circuit" over inputs x, w

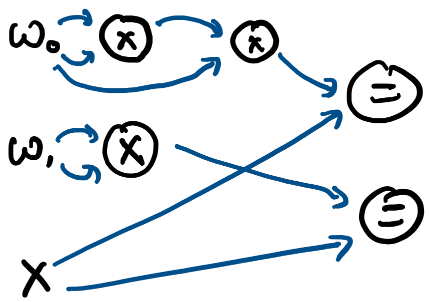
Arithmetic Circuits (ACs), Part I

- Domain: "prime field"
  - p: a large (~255 bit) prime
  - $\mathbb{Z}_p$: the integers, mod p
    - operations: $+, \times, =$ (mod p)
  - Example in $\mathbb{Z}_5$:
    - $4 + 5 = 9 = 4$
    - $4 \times 4 = 16 = 1$

- ACs as systems of field equations:
  - Example:
    - $w_0 \times w_0 \times w_0 = x$
    - $w_1 \times w_1 = x$
    - Addition is also OK

Arithmetic Circuits (ACs), Part II

- ACs as circuits
  - Directed, acyclic graph
  - Nodes: inputs, gates, constants
  - Edges: wires/connections
- Example:
  - $w_0 \times w_0 \times w_0 = x$
  - $w_1 \times w_1 = x$
- As a circuit:



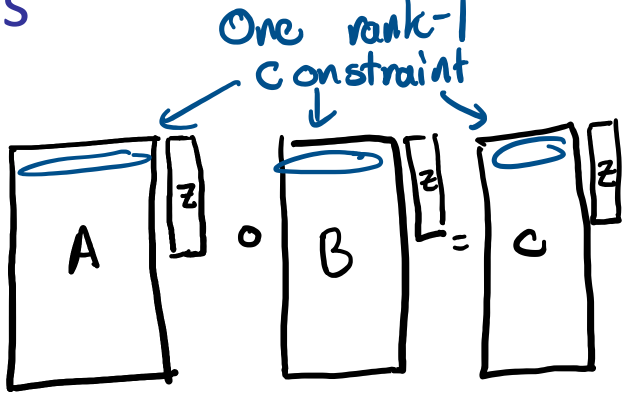
R1CS: a common Arithmetic Circuit format

- R1CS: format for ZKP ACs
- Definition:
  - x: field elements $x_1, \ldots, x\_\ell$
  - w: $w_1, \ldots, w_{m-\ell-1}$
  - ϕ: n equations of form
    - $\alpha \times \beta = \gamma$
    - where α, β, γ are affine combinations of variables
- Examples:
  - $w_2 \times (w_3 - w_2 - 1) = x_1$
  - $w_2 \times w_2 = w_2$
  - ~~$w_2 \times w_2 \times w_2 = x_1$~~
    - $w_2 \times w_2 = w_4$
    - $w_4 \times w_2 = x_1$

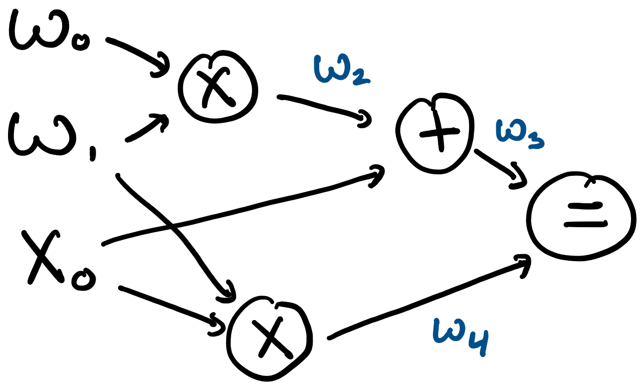
R1CS: Matrix Definition

- x: vector of ℓ field elements
- w: vector of $m - \ell - 1$ field elements
- ϕ: matrices $A, B, C \in \mathbb{Z}_p^{n \times m}$
  - $z = (1 \parallel x \parallel w) \in \mathbb{Z}_p^m$
  - Holds when $Az \circ Bz = Cz$

↑ element-wise product

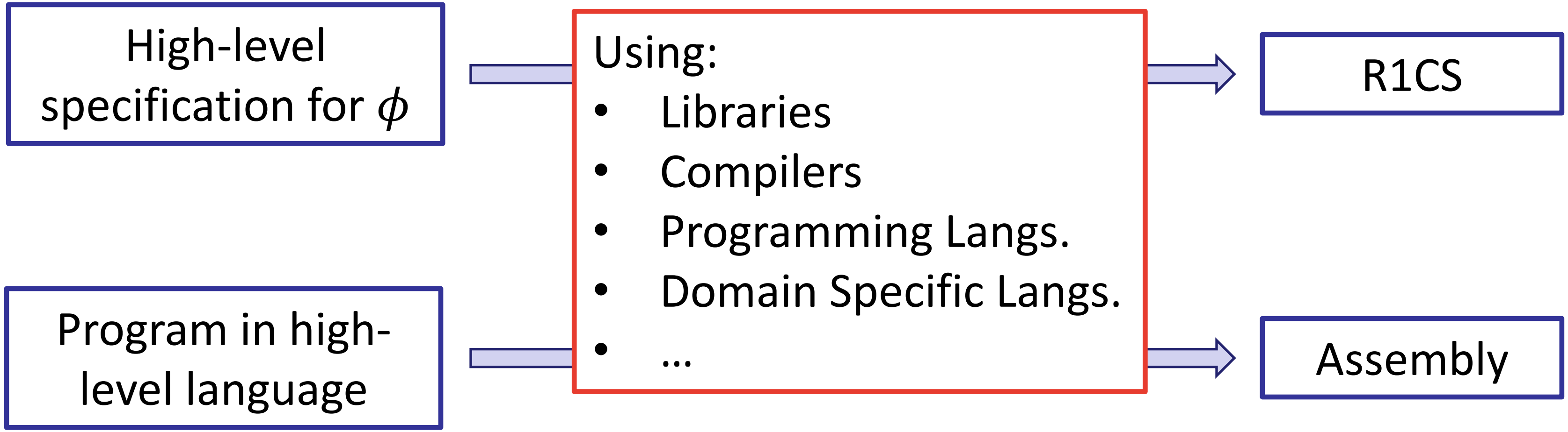


Writing an AC as R1CS (Example)



- Step 1: intermediate *ws*
- Step 2: write equations
  - $w_0 \times w_1 = w_2$
  - $w_3 = w_2 + x_0$
  - $w_1 \times x_0 = w_4$
  - $w_3 = w_4$

Zooming out: a Programming Languages problem



The Idea

High-level code ⇨ Compiler/Library ⇨ R1CS ⇨ ZK Proof System

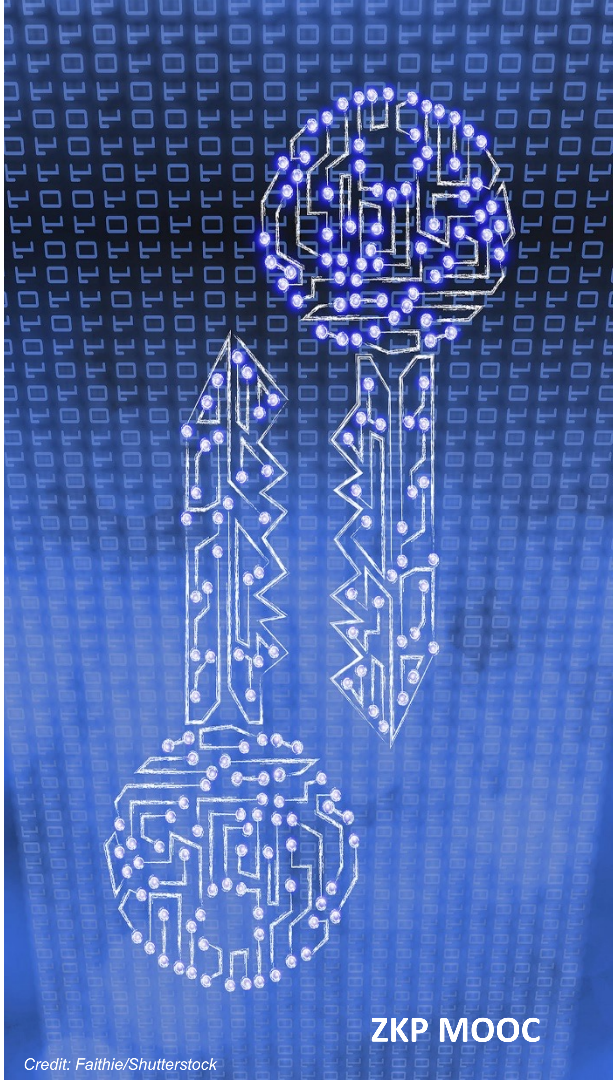
- Booleans
- Structures
- Modules
- Functions
- …

An Example

Zcash Circuit ⇨ Bellman Library ⇨ R1CS ⇨ Groth16

- Merkle tree
- Pedersen Hash
- Signatures
- Spend logic
- …

An HDL for R1CS



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Programming Languages (PLs) vs. Hardware Description Languages (HDLs)

PL objects
- Variables
- Operations
- Program/Functions

HDL objects
- Wires
- Gates
- Circuit/Sub-circuits

PL actions
- Mutate variables
- Call functions

HDL actions
- Connect wires
- Create sub-circuits

HDLs: From Digital to Arithmetic

HDLs for Digital Circuits

- Verilog
- SystemVerilog
- VHDL
- Chisel
- …

An HDL for R1CS

- circom
  - wires: R1CS variables
  - gates: R1CS constraints
- a circom circuit does 2 things:
  - sets variable values
  - creates R1CS constraints

Circom: Base Language

- A "template" is a (sub)circuit
- A "signal" is a wire
  - "input" or "output"
- "<--" sets signal values
- "===" creates constraints
  - Must be rank-1:
    - one side: linear
    - other side: quadratic
- "<==" does both

```
template Multiply() {
  signal input x;
  signal input y;
  signal output z;

  z <-- x * y;
  z === x * y;
  // ERROR: z === x * x * y
  // OR   : z <== x * y;
}

component main {public [x]} =
  Multiply();
```

Verifier knows

Circom: Metaprogramming Language

- Template arguments
- Signal arrays
- Variables
  - Mutable
  - Not signals
  - Evaluated at compile-time
- Loops
- If statements
- Array accesses

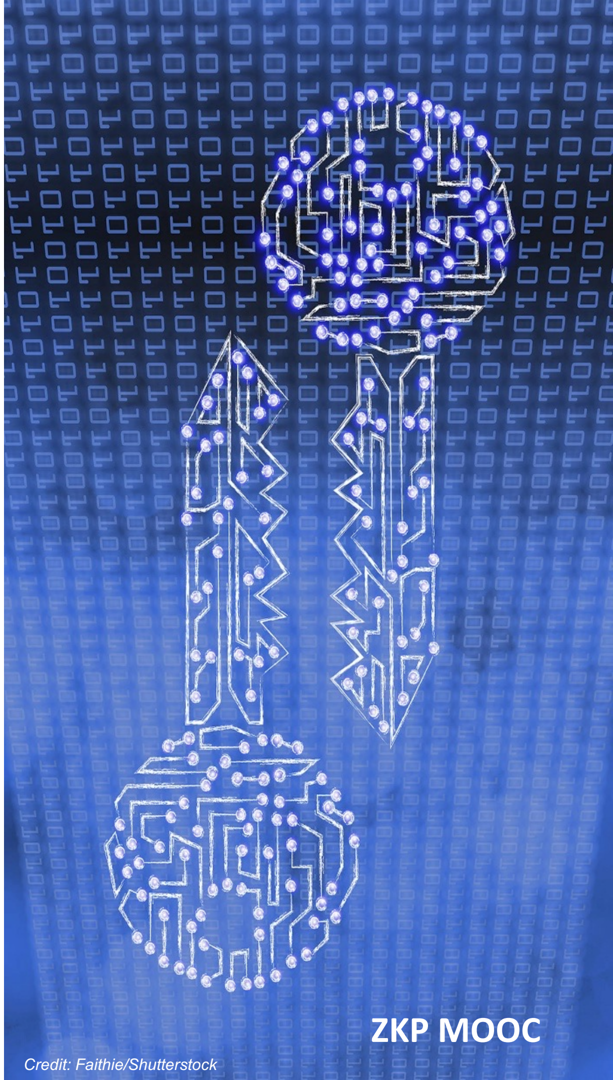
```
template RepeatedSquaring(n) {
  signal input x;
  signal output y;

  signal xs[n];
  xs[0] <== x;
  for (var i = 0; i <= n; i++) {
    xs[i+1] <== xs[i] * xs[i];
  }
  y <== xs[n];
}
component main {public [x]} =
  RepeatedSquaring(1000);
```

Circom: Witness Computation & Sub-circuits

- Witness computation: more general than R1CS
  - "<--" is more general than "==="
- "component"s hold sub-circuits
  - Access inputs/outputs with dot-notation

```
template NonZero() {
  signal input in;
  signal inverse;
  inverse <-- 1 / in; // not R1CS
  1 === in * signal;   //  is R1CS
}

template Main() {
  signal input a; signal input b;
  component nz = NonZero();
  nz.in <== a;
  0 === a * b;
}
```

Circom Tutorial



Tutorial Example: Sudoku

- 9 by 9 grid
- Some cells have #s
- Goal: fill all cells with 1…9
- Rule: no duplicates in any:
  - ~~Column~~
  - Row
  - ~~3x3 sub-grid~~

} ϕ

	7	5		9				6
	2	3		8			4	
8					3			1
5			7		2			
	4		8		6		2	
			9		1			3
9			4					7
	6			7		5	8	
7				1		3	9	

Puzzle

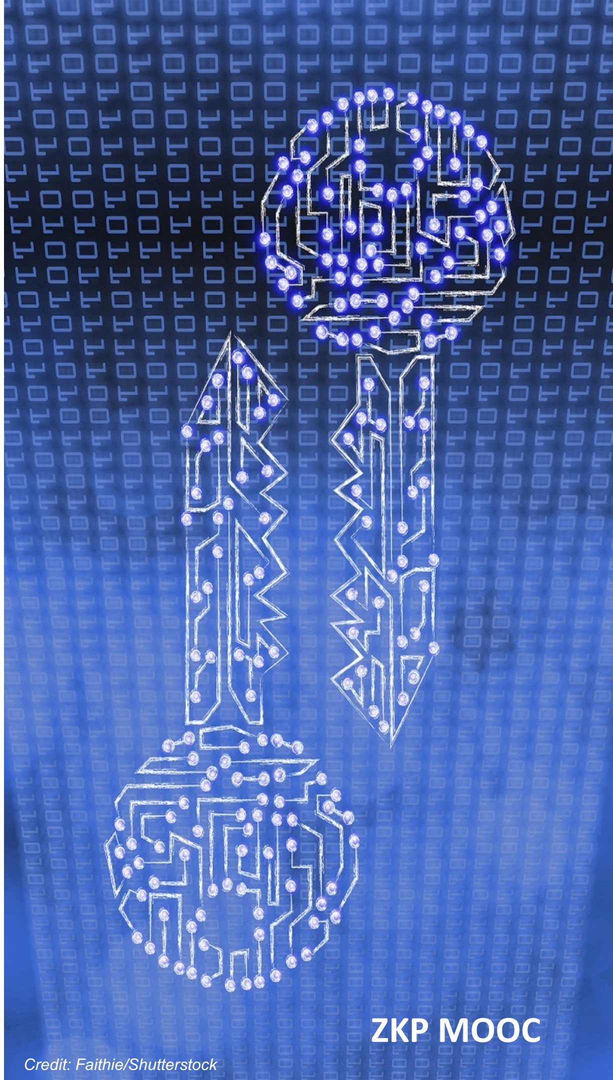
x

1	7	5	2	9	4	8	3	6
6	2	3	1	8	7	9	4	5
8	9	4	5	6	3	2	7	1
5	1	9	7	3	2	4	6	8
3	4	7	8	5	6	1	2	9
2	8	6	9	4	1	7	5	3
9	3	8	4	2	5	6	1	7
4	6	1	3	7	9	5	8	2
7	5	2	6	1	8	3	9	4

Solution

ω

A Library for R1CS

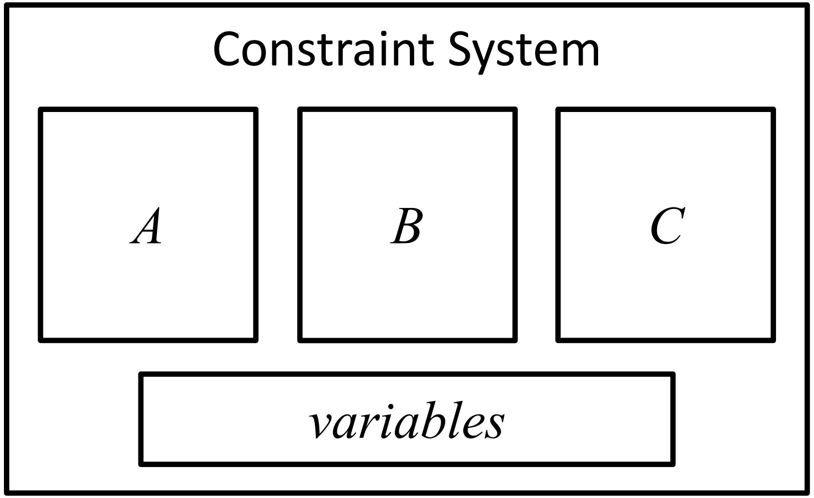


Circom: Recap

- An HDL for R1CS
- Key features:
  - Direct control over constraints
  - Custom language
    - Can be good
    - Can be bad

R1CS Libraries

- A library in a *host* language (Eg: Rust, OCaml, C++, Go, …)
- Key type: *constraint system*
  - Maintains state about R1CS constraints and variables
- Key operations:
  - create variable
  - create *linear combinations* of variables
  - add constraint



ConstraintSystem Operations

Variable creation

`cs.add_var(p, v) → id`

- `cs:` constraint system
- `p:` visibility of variable
- `v:` assigned value
- `id:` variable handle

Linear Combination creation

`cs.zero()` : returns the empty LC

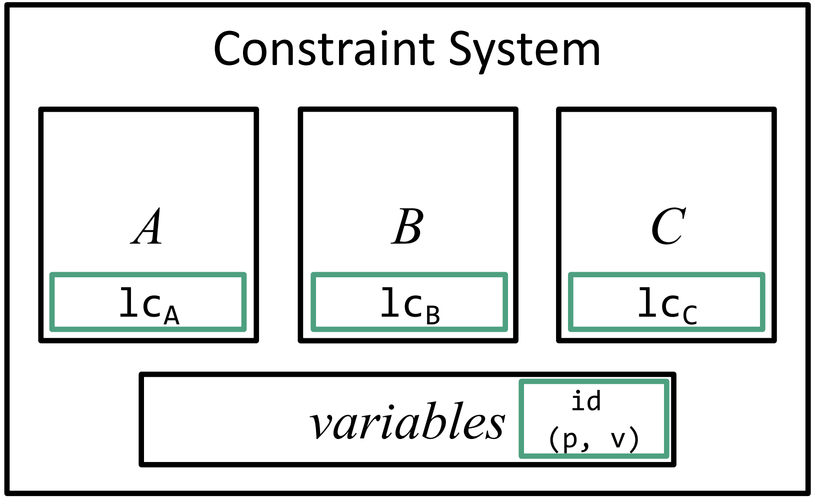
`lc.add(c, id) → lc'`

- `id:` variable
- `c:` coefficient
- `lc' := lc + c * id`

Adding constraints

$\texttt{cs.constrain}(\texttt{lc}_A, \texttt{lc}_B, \texttt{lc}_C)$

- Adds a constraint $\texttt{lc}_A \times \texttt{lc}_B = \texttt{lc}_C$



Example: Boolean AND

```
fn and(cs: ConstraintSystem, a: Var, b: Var) → Var {
    let result = cs.new_witness_var(|| a.value() & b.value());
    self.cs.enforce_constraint(
        lc!() + a,
        lc!() + b,
        lc!() + result,
    );
    result
}
```

Create result variable

Enforce constraint

Create linear combinations

Example: Boolean AND

Create
var

```
fn and(cs:
    let re                                        .value());
    self.c
        lc
        lc
        lc
    );
    result
}
```

This is unpleasant, tedious, and error-prone!

Can you imagine writing a complex algorithm like signature verification in this style?

Idea: Leverage Language Abstractions!

We can use language abstractions like structs, operator [...] ethods, etc. to allow better developer UX:

Wrap variable in dedicated type

```
struct Boolean { var: Var };

impl BitAnd for Boolean {
    fn and(self: Boolean, other: Boolean) → Boolean {
        // Same as before
        Boolean { var: result }
    }
}
```

Implement interface for operator overloading

Does it work? Yes!

Can use abstractions like normal code:

```
let a = Boolean::new_witness(|| true);
let b = Boolean::new_witness(|| false);
(a & b).enforce_equal(Boolean::FALSE);
```

Many different gadget libraries:

- libsnark: gadgetlib (C++)
- arkworks: r1cs-std + crypto-primitives (Rust)
- Snarky (Ocaml)
- Gnark (Go)

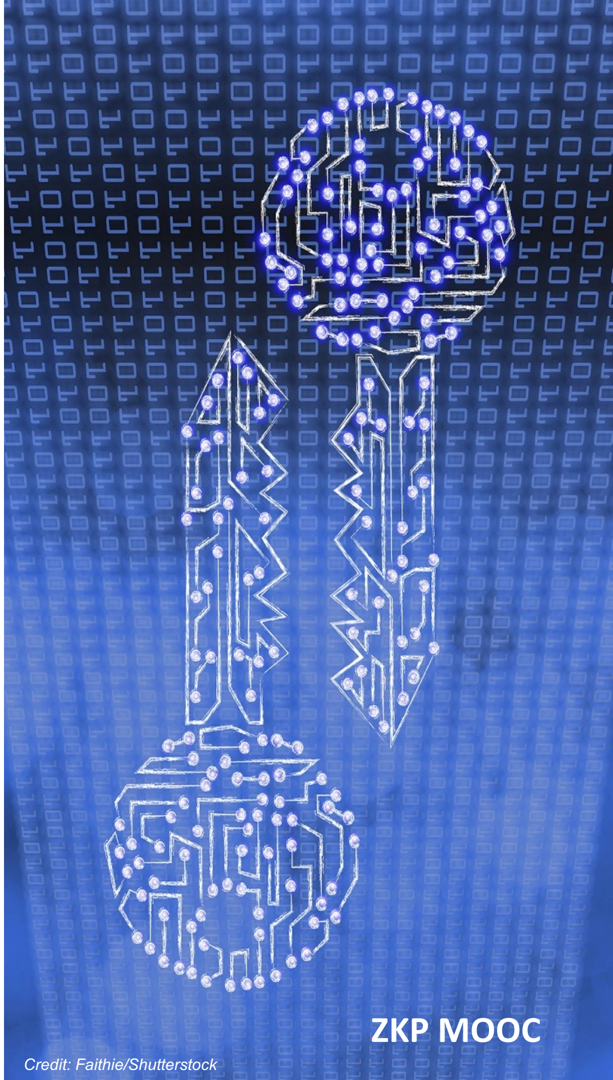
What about Witness Computation?

- Can perform arbitrary computations to generate witnesses

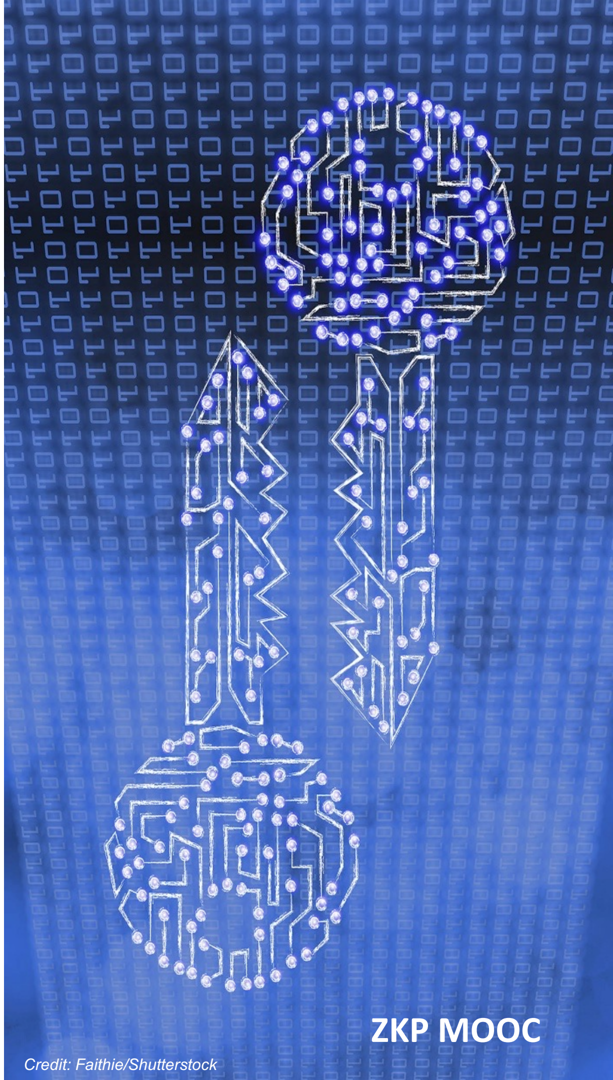
Closure (lambda) executed only during proving!

```
let a = Boolean::new_witness(|| (4 == 5) & (x < y));
let b = Boolean::new_witness(|| false);
(a & b).enforce_equal(Boolean::FALSE);
```

Arkworks Tutorial



Compiling a Programming Language to R1CS



HDLs & Circuit Libraries

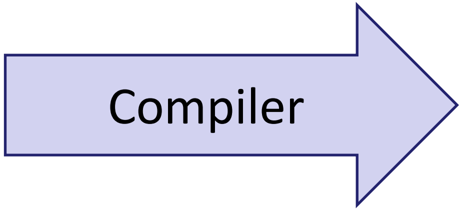
- Difference:
  - Host language v. custom language
- Similarities:
  - explicit wire creation (explicitly wire values)
  - explicit constraint creation
- Do we need to explicitly build a circuit?
  - No!

Compiling PLs to Circuits (Idea)

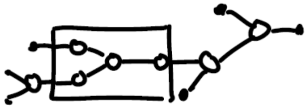
Program

```
fn main(...){
    ...
}
```

- Variables
- Mutation
- Functions
- Arrays



R1CS



- Wires
- Constraints

ZoKrates: Syntax

- Struct syntax for custom types
- Variables contain values during execution/proving
- Can annotate privacy
- "assert" creates constraints

```
type F = field;
```

Verifier knows

```
def main(public F x, private
F[2] ys) {
    field y0 = y[0];
    field y1 = y[1];
    assert(x = y0 * y1);
}
```

Zokrates: Language features

- Integer generics
- Arrays
- Variables
  - Mutable
- Fixed-length loops
- If expressions
- Array accesses

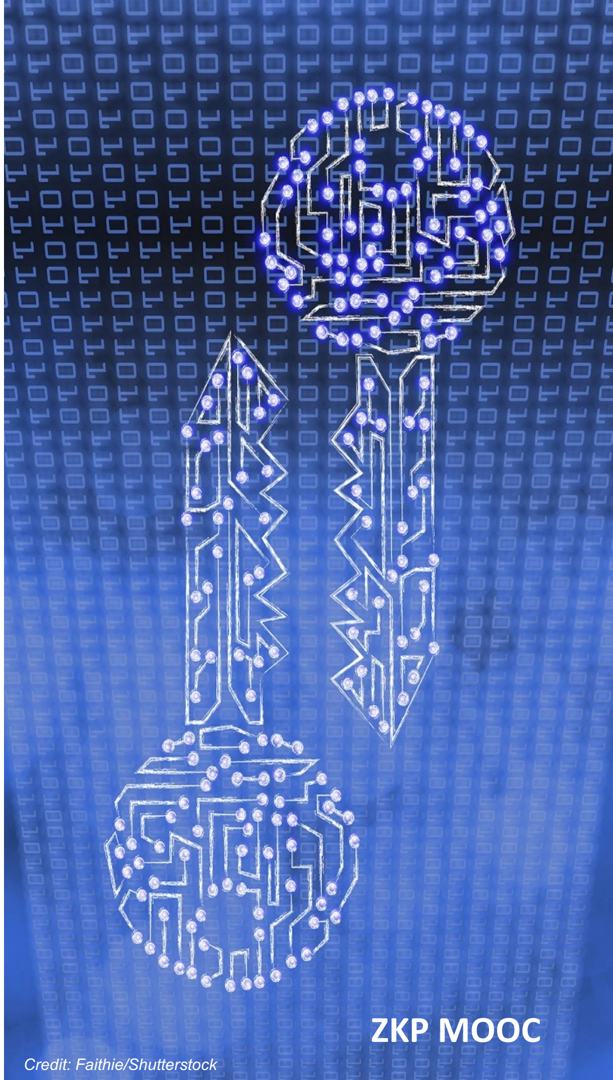
```
def repeated_squaring<N>(field x) -> field {
    field[N] mut xs;
    xs[0] = x;
    for u32 i in 0..n {
        xs[i + 1] = xs[i] * xs[i];
    }
    return xs[N];
}

def main (public field x) -> field {
    repeated_squaring::<1000>(x)
}
```

What about Witness Computation?

- No way to compute witnesses
- All witnesses must be provided as input

```
def main(private field a, public
field b) {
    assert(a * b == 1)
}
```

ZoKrates Tutorial



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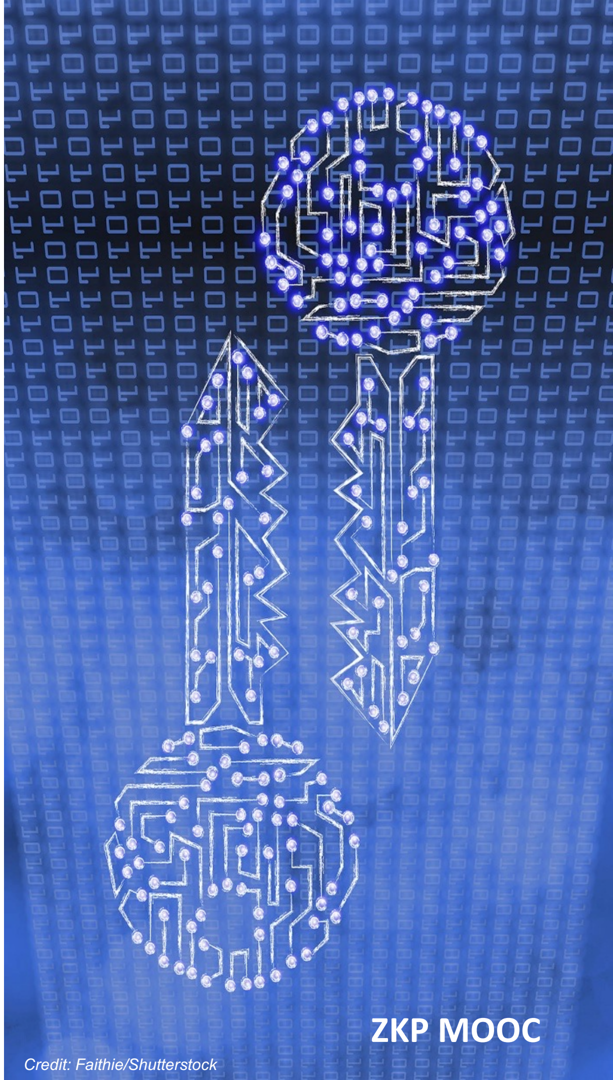
ZKP Toolchains:
A Quick Tour



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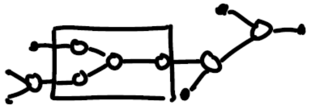
Toolchain Type

<u>HDL</u>

a language for describing circuit synthesis



<u>Library</u>

a library for describing circuit synthesis

```
circ.add_wire(...)
circ.add_gate(...)
```

<u>PL + Compiler</u>

a language, compiled to a circuit

```
fn main(...){
    ...
}
```

Toolchain Types, Organized

Standalone Language?

Language Type		No	Yes
	Circuit	Library (arkworks)	HDL (circom)
	Program		PL (noir)

circom

Pros:

- Clear constraints
- Elegant syntax

Cons:

- Hard to learn
- Limited abstraction

===

HDL

types?

arkworks

Pros:

- Clear constraints
- As expressive as Rust

Cons:

- Need to know Rust
- Few optimizations

manual opts

ZoKrates

Pros:

- Easiest to learn
- Elegant syntax

Cons:

- Limited witness computation

just a PL

always implicit

Other toolchains

<u>HDL</u>

- Circom

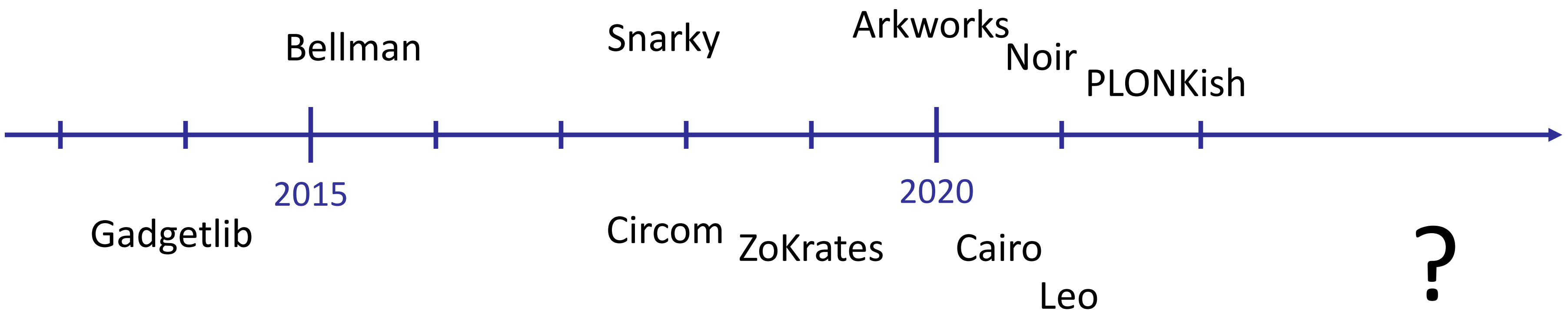
<u>Library</u>

- Arkworks (Rust)
- Gadgetlib (C++)
- Bellman (Rust)
- Snarky (OCaml)
- PLONKish (Rust)

<u>PL + Compiler</u>

- ZoKrates
- Noir
- Leo
- Cairo

Timeline


Bellman
Snarky
Arkworks
Noir
PLONKish
2015
2020
Gadgetlib
Circom
ZoKrates
Cairo
Leo
?

Shared Compiler Infrastructure?

Source

Cairo
ZoKrates
Circom
PLONKish
Noir
Snarky
Bellman
Gadgetlib
Leo

Common Techniques

RAM-
Bo
Fixed-
Opti

ures
on
es
Flow

CirC

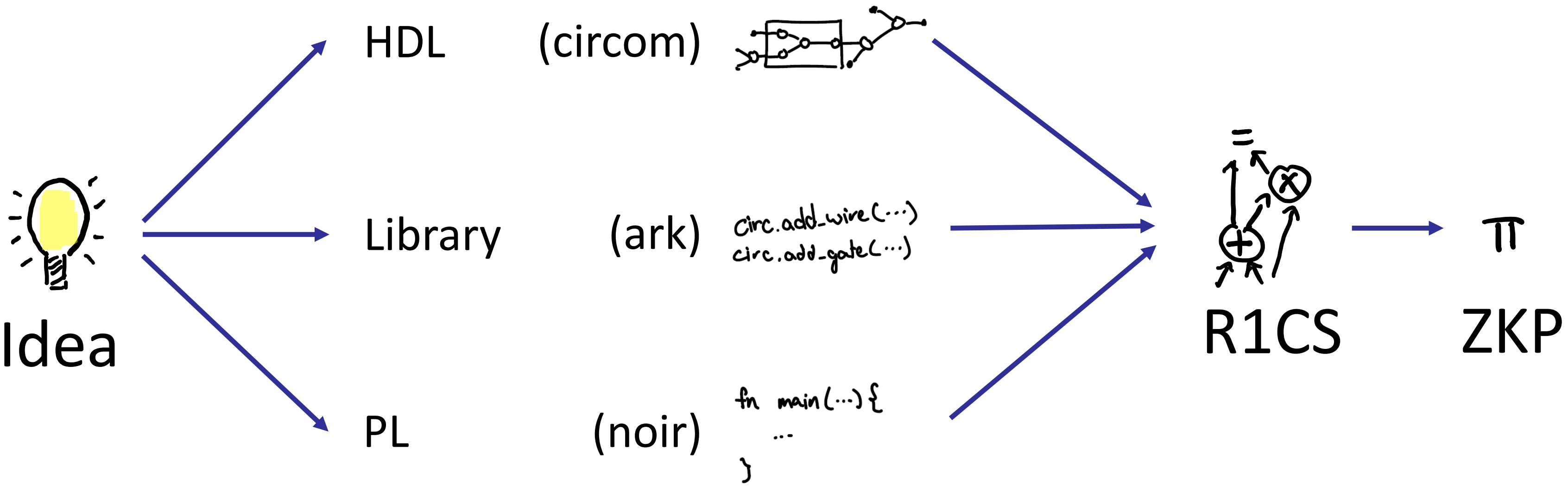
A library for
ZKP languages

github.com/circify/circ

Target

R1CS
Plonk
AIR

Summary


HDL
(circom)
Library
(ark)
circ.add_wire(...)
circ.add_gate(...)
PL
(noir)
fn main(...){
...
}
Idea
R1CS
ZKP
π

End of Lecture